

Dark energy and particle mixing

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We show that the vacuum condensate due to particle mixing is responsible of a dynamically evolving dark energy. In particular, we show that values of the adiabatic index close to -1 for vacuum condensates of neutrinos and quarks imply, at the present epoch, contributions to the vacuum energy compatible with the estimated upper bound on the dark energy.

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I. INTRODUCTION

The experimental achievements proving neutrino oscillations [1, 2] seem to indicate a promising path beyond the Standard Model of electro-weak interaction for elementary particles. On the other hand, an increasing bulk of data has been accumulated in the last few years paving the way to the emergence of a new standard cosmological model usually referred to as the *concordance model*. The Hubble diagram of Type Ia Supernovae (SNeIa), measured by both the Supernova Cosmology Project [3] and the High- z Team [4] up to redshift $z \sim 1$, was the first evidence that the universe is undergoing a phase of accelerated expansion. Balloon born experiments, such as BOOMERanG [5] and MAXIMA [6], determined the location of the first and second peak in the anisotropy spectrum of cosmic microwave background radiation (CMBR) pointing out that the geometry of the universe is spatially flat. If combined with constraints coming from galaxy clusters on the matter density parameter Ω_M , these data indicate that the universe is dominated by a non-clustered fluid with negative pressure, generically referred to as *dark energy*, which is able to drive the accelerated expansion. This picture has been further strengthened by the more precise measurements of the CMBR spectrum, due to the WMAP experiment [7], and by the extension of the SNeIa Hubble diagram to redshifts higher than 1 [8]. Several models trying to explain this phenomenon have been presented; the simplest explanation is claiming for the well known cosmological constant Λ [9]. Although the best fit to most of the available astrophysical data [7], the Λ CDM model fails in explaining why the inferred value of Λ is so tiny (123 orders of magnitude lower) compared to the typical vacuum energy values predicted by particle physics and why its energy density is today comparable to the matter density (the so called *coincidence problem*).

In this paper we study the possibility that a link between high energy physics and cosmology might be found in the mechanism of particle mixing. In our discussion we resort to previous investigations which led us to the conclusion that neutrino mixing might contribute to the dark energy budget of the universe [10, 11]. We show that the vacuum condensate due to particle mixing is responsible of a dynamically evolving dark energy. In particular, we show that values of the adiabatic index close to -1 , both for vacuum condensates of neutrinos and quarks imply, at the present epoch, contributions to the vacuum energy compatible with the observed cosmological constant. We compute such a value and show that the condensate could give rise also to the dark matter component of the Universe, besides the accelerating one. Our discussion and conclusions rest on the QFT formalism for particle mixing, which has been extensively discussed in recent years in the literature [12, 13, 14, 15, 16, 17, 18]. For the reader convenience we summarize it in the Appendix A.

The fact that the mixing phenomenon might be a source for the dark energy appears to be relevant from a genuine experimental point of view since, up to now, none of the exotic candidates for dark matter and dark energy, has been detected at a fundamental level.

The layout of the paper is the following. In Section II we present the particle mixing condensate in the early and in the present epoch. In Section III we compute the fermion mixing contributions to the dark energy at the present epoch. Conclusions are drawn in Section IV. We outline the QFT formalism for fermion mixing in the Appendix A. In the Appendix B are reported useful computations.

II. PARTICLE MIXING AND DARK ENERGY

As mentioned above, experimental data indicate that the today observed universe can be described as an accelerating Hubble fluid where the contribution of dark energy component to the total matter-energy density is $\Omega_\Lambda \simeq 0.7$.

Moreover, the cosmic flow is "today" accelerating while it was not so at intermediate redshift z (e.g. $1 < z < 10$) where large scale structures have supposed to be clustered. Thus, physically motivated cosmological models should undergo, at least, three phases: an early accelerated inflationary phase, an intermediate standard matter dominated (decelerated) phase and a final, today observed, dark energy dominated (accelerated) phase. This means that we have to take into account some form of *dark energy* which evolves from early epochs inducing the today observed acceleration.

In this Section we show that the energy density due to the vacuum condensate arising from particle mixing can be interpreted as an evolving dark energy. The calculation here presented is performed for Dirac fermion fields in a Minkowski space-time. It can be extended to curved space-times, as it will be shown in a forthcoming work.

Let us calculate the contributions ρ_{vac}^{mix} and p_{vac}^{mix} of the particle mixing to the vacuum energy density and to the vacuum pressure. Such a contributions are given respectively by the $(0,0)$ and (j,j) components of the energy-momentum tensor of the condensed particles given in Eqs.(A21)-(A23) in Appendix A.

The energy-momentum tensor density $\mathcal{T}_{\mu\nu}(x)$ for the fermion fields ψ_i , $i = 1, 2, 3$ [20], is

$$:\mathcal{T}_{\mu\nu}(x) := \frac{i}{2} : \left(\bar{\Psi}_m(x) \gamma_\mu \overleftrightarrow{\partial}_\nu \Psi_m(x) \right) : \quad (1)$$

where $\Psi_m = (\psi_1, \psi_2, \psi_3)^T$ and the normal ordering is with respect to the vacuum $|0\rangle_m$ for the massive fields. Then the energy momentum tensor density of the vacuum condensate is given by

$$\mathcal{T}_{\mu\nu}^{cond}(x) = {}_f\langle 0(t) | : \mathcal{T}_{\mu\nu}(x) : | 0(t) \rangle_f, \quad (2)$$

where $|0(t)\rangle_f$ is the vacuum for the flavor fields (see Appendix A).

A. Early universe epochs

In the early universe epochs, when the breaking of the Lorentz invariance of the vacuum is not negligible, ρ_{vac}^{mix} presents also space-time dependent condensate contributions. This implies that the contribution ρ_{vac}^{mix} of the particle mixing to the vacuum energy density is given by computing the expectation value of the $(0,0)$ component of the energy-momentum tensor $:T_{00} := \int d^3x : \mathcal{T}_{00}(x) :$ in the physical vacuum $|0(t)\rangle_f$:

$$\rho_{vac}^{mix} \equiv \frac{1}{V} {}_f\langle 0(t) | : T^{00}(0) : | 0(t) \rangle_f. \quad (3)$$

$:T_{00}:$ is for definition the Hamiltonian $:H:$ in Eq.(A24) that, in terms of the annihilation and creation operators of ψ_1 , ψ_2 and ψ_3 , is

$$:T_{00} := \sum_i \sum_r \int d^3\mathbf{k} \omega_{k,i} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r \right). \quad (4)$$

The notation is the one introduced in the Appendix (Eq. (A3)). Note that T_{00} is time independent, moreover, within the QFT mixing formalism we have

$${}_f\langle 0 | :T_{00} : | 0 \rangle_f = {}_f\langle 0(t) | :T_{00} : | 0(t) \rangle_f \quad (5)$$

for any t . We then obtain

$$\rho_{vac}^{mix} = \sum_{i,r} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \omega_{k,i} \left({}_f\langle 0 | \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r | 0 \rangle_f + {}_f\langle 0 | \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r | 0 \rangle_f \right),$$

which, introducing the cut-off K , becomes

$$\begin{aligned} \rho_{vac}^{mix} = & \frac{2}{\pi} \int_0^K dk k^2 \left[\omega_{k,1} \left(s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2 \right) + \omega_{k,2} \left(|-s_{12} c_{23} + e^{i\delta} c_{12} s_{23} s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2 \right) \right. \\ & \left. + \omega_{k,3} \left(|-c_{12} s_{23} + e^{i\delta} s_{12} c_{23} s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12} s_{23} + e^{i\delta} c_{12} c_{23} s_{13}|^2 |V_{13}^{\mathbf{k}}|^2 \right) \right]. \end{aligned} \quad (6)$$

Here $\omega_{k,i} = \sqrt{k^2 + m_i^2}$ and the notation is the one introduced in the Appendix A for the CKM matrix elements (Eq. (A2)) and for the Bogoliubov coefficients $V_{ij}^{\mathbf{k}}$ (Eqs. (A18) and Eqs. (A19)). In any epoch, the energy density induced by the particle mixing condensate can be expressed as

$$\rho_{vac}^{mix} = T_{vac}^{mix} + V_{vac}^{mix} \quad (7)$$

where the kinetic term T_{vac}^{mix} and the potential term V_{vac}^{mix} are respectively given by

$$T_{vac}^{mix} = \frac{2}{\pi} \int_0^K dk k^2 \left[\frac{k^2}{\omega_{k,1}} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) + \frac{k^2}{\omega_{k,2}} (|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2) \right. \\ \left. + \frac{k^2}{\omega_{k,3}} (|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2) \right], \quad (8)$$

and

$$V_{vac}^{mix} = \frac{2}{\pi} \int_0^K dk k^2 \left[\frac{m_1^2}{\omega_{k,1}} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) + \frac{m_2^2}{\omega_{k,2}} (|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2) \right. \\ \left. + \frac{m_3^2}{\omega_{k,3}} (|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2) \right]. \quad (9)$$

Eqs.(8) and (9) are obtained from Eq.(6) by using the relation $\omega_{k,i} = \frac{k^2}{\omega_{k,i}} + \frac{m_i^2}{\omega_{k,i}}$.

In a similar way, the contribution p_{vac}^{mix} of particle mixing to the vacuum pressure is given by the expectation value of $:T_{jj}:$ (where no summation on the index j is intended) in the vacuum $|0(t)\rangle_f$:

$$p_{vac}^{mix} = -\frac{1}{V} \eta_{jj} :T^{jj}: |0(t)\rangle_f. \quad (10)$$

Being

$$:T^{jj}:= \sum_i \sum_r \int d^3\mathbf{k} \frac{k^j k^j}{\omega_{k,i}} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right), \quad (11)$$

in the case of the isotropy of the momenta we have $T^{11} = T^{22} = T^{33}$, then

$$p_{vac}^{mix} = \frac{2}{3\pi} \int_0^K dk k^2 \left[\frac{k^2}{\omega_{k,1}} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) + \frac{k^2}{\omega_{k,2}} (|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2) \right. \\ \left. + \frac{k^2}{\omega_{k,3}} (|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2) \right]. \quad (12)$$

From Eqs.(6) and (12), we define the adiabatic index $w^{mix} \equiv p_{vac}^{mix}/\rho_{vac}^{mix}$. The plot of w^{mix} as function of the momentum cut-off K (Fig.1) shows that $w^{mix} = 1/3$ when the cut-off is chosen to be $K \gg \bar{m}$ where \bar{m} is the largest of m_1, m_2, m_3 and w^{mix} goes to zero for $K \leq \sqrt[3]{m_1 m_2 m_3}$.

This means that the condensate "mimics" the behavior of a perfect fluid of dust and radiation at the extreme values of the cut-off. From a dynamical point of view, it behaves as radiation in the relativistic regime ($w^{mix} \simeq 1/3$) and as dark matter in the non-relativistic regime ($w^{mix} \simeq 0$). Thus, in the early Universe and in the regions in which the breaking of Lorentz invariance of the vacuum is not negligible, the condensate could give rise to the dark matter component of the Universe.

We note that according to this result, at the early universe epoch, the particle mixing condensate does not give contributions to the "standard" dark energy (the adiabatic index w^{mix} assumes, as we said, values in the range $0 \leq w^{mix} \leq 1/3$).

This gives the possibility to achieve the large scale structure formation as requested in a standard matter-radiation dominated regime and is in complete agreement with the WMAP results [21]. Indeed, microwave light seen by WMAP from when the universe was only 380.000 years old, shows that, at that time, neutrinos made up 10% of the universe, atoms 12%, dark matter 63%, photons 15%, and dark energy was negligible. In contrast, estimates from WMAP data show the current universe consists of 4.6% of atoms, 23% dark matter, 72% dark energy and less than 1 percent neutrinos.

The values of ρ_{vac}^{mix} and p_{vac}^{mix} which we obtain are time-independent since, as said, we are taking into account the Minkowski metric. Considering a curved space-time, time-dependence has to be taken into account but the essence of the result is expected to be the same (work is in progress on such an issue).

B. Universe at present epoch

At the present epoch, the breaking of the Lorentz invariance of the vacuum is very small and then ρ_{vac}^{mix} comes almost completely from space-time independent condensate contributions (i.e. the contributions to the energy density

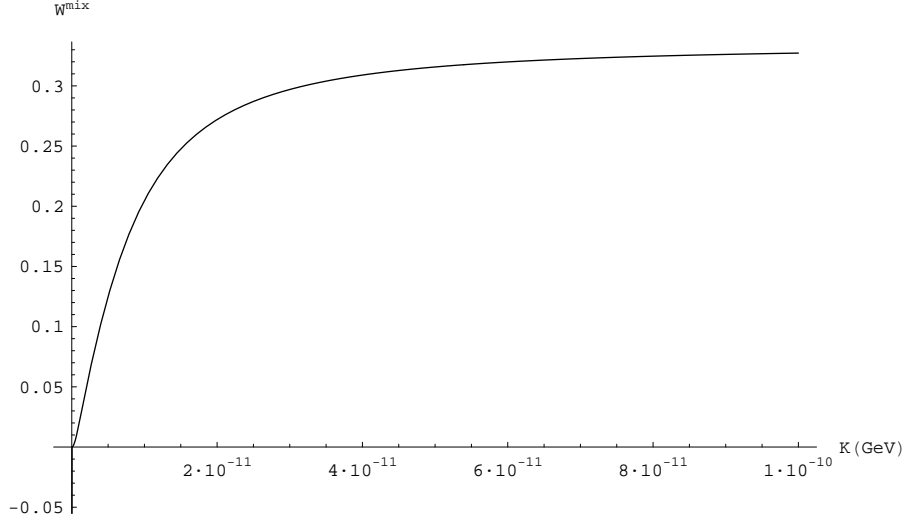


Figure 1: The adiabatic index w^{mix} as a function of cut-off K .

of the vacuum $|0\rangle_f$ for mixed fields carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$ can be neglected). Then, in a flat space-time the kinetic term T_Λ^{mix} is negligible with respect to the potential ones V_Λ^{mix} : $T_\Lambda^{mix} \ll V_\Lambda^{mix}$ and the energy-momentum density tensor of the vacuum condensate is approximatively given by

$$\mathcal{T}_{\mu\nu}^{cond} \simeq \eta_{\mu\nu} \sum_i m_i \int \frac{d^3x}{(2\pi)^3} {}_f\langle 0 | : \bar{\psi}_i(x) \psi_i(x) : | 0 \rangle_f = \eta_{\mu\nu} \rho_\Lambda^{mix}. \quad (13)$$

Since in a homogeneous and isotropic universe, the energy-momentum tensor density of the vacuum condensate can be written as $\mathcal{T}_{\mu\nu}^{cond} = diag(\rho_\Lambda^{mix}, p_\Lambda^{mix}, p_\Lambda^{mix}, p_\Lambda^{mix})$, by equating this expression with Eq.(13) and using $\eta_{\mu\nu} = diag(1, -1, -1, -1)$, we obtain the state equation: $\rho_\Lambda^{mix} \simeq -p_\Lambda^{mix}$, consistently with the vacuum Lorentz invariance.

This means that the vacuum condensate, coming from particle mixing, contributes today to the dynamics of the universe with a cosmological constant behavior [11]. ρ_Λ^{mix} computed from Eq.(13) thus turns out to be

$$\begin{aligned} \rho_\Lambda^{mix} = & \frac{2}{\pi} \int_0^{K_\Lambda} dk k^2 \left[\frac{m_1^2}{\omega_{k,1}} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) + \frac{m_2^2}{\omega_{k,2}} \left(|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2 \right) \right. \\ & \left. + \frac{m_3^2}{\omega_{k,3}} \left(|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2 \right) \right], \end{aligned} \quad (14)$$

which can be written as

$$\begin{aligned} \rho_\Lambda^{mix} = & \frac{2}{\pi} \int_0^{K_\Lambda} dk k^2 \left\{ \frac{m_1^2}{\omega_{k,1}} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) + \frac{m_2^2}{\omega_{k,2}} [(s_{12}^2 c_{23}^2 + c_{12}^2 s_{23}^2 s_{13}^2) |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2] \right. \\ & \left. + \frac{m_3^2}{\omega_{k,3}} [(c_{12}^2 s_{23}^2 + s_{12}^2 c_{23}^2 s_{13}^2) |V_{23}^{\mathbf{k}}|^2 + (s_{12}^2 s_{23}^2 + c_{12}^2 c_{23}^2 s_{13}^2) |V_{13}^{\mathbf{k}}|^2] \right\} \\ & - \frac{4}{\pi} s_{12} c_{23} c_{12} s_{23} s_{13} c_\delta \int_0^{K_\Lambda} dk k^2 \left\{ \frac{m_2^2}{\omega_{k,2}} |V_{12}^{\mathbf{k}}|^2 + \frac{m_3^2}{\omega_{k,3}} [|V_{23}^{\mathbf{k}}|^2 - |V_{13}^{\mathbf{k}}|^2] \right\}, \end{aligned} \quad (15)$$

where $c_\delta = \cos \delta$. Note that ρ_Λ^{mix} also depends on the CP violating phase δ .

We observe that the value of the integral is conditioned by the appearance in the integrand of the $|V_{ij}^{\mathbf{k}}|^2$ factors. The integral, and thus ρ_Λ^{mix} , would be zero for $|V_{ij}^{\mathbf{k}}|^2 = 0$ for any $|\mathbf{k}|$, as it is in the quantum mechanical (Pontecorvo) formalism [22, 23, 24, 25]. In the present QFT formalism the $|V_{ij}^{\mathbf{k}}|^2$'s account for the vacuum condensate (Eqs. (A21) - (A23)) and $|V_{ij}^{\mathbf{k}}|^2$ goes to zero only for large momenta, getting its maximum value for $|\mathbf{k}| \approx \sqrt{m_i m_j}$ for any $i, j = 1, 2, 3$ [16].

Proceeding in our calculation, we obtain that the integral (14) diverges in K_Λ as $m_i^4 \log(2K_\Lambda/m_j)$, with $i, j = 1, 2, 3$ (see Appendix B). One also sees that $\frac{d\rho_\Lambda^{mix}(K_\Lambda)}{dK_\Lambda} \propto \frac{1}{K_\Lambda} \rightarrow 0$ for large K_Λ . An interesting question to ask is how the

result $\rho_\Lambda^{mix} \propto m_i^4 \log(2K_\Lambda/m_j)$, directly obtained in our approach, is related to the conjecture [26] that the small value of the cosmological constant $\rho_\Lambda \propto (10^{-3}eV)^4$ is associated with the vacuum in a theory which has a fundamental mass scale $m \sim 10^{-3}eV$.

III. PARTICLE MIXING CONDENSATE CONTRIBUTIONS AT THE PRESENT EPOCH

In this Section we find a constraint on the cut-off on the momenta, at the present epoch, and we derive an expression of the adiabatic index of the particle condensates, $w_\Lambda^{mix} = p_\Lambda^{mix}/\rho_\Lambda^{mix}$, as function of the cut-off. Then we show that values of the adiabatic index close to -1 , both for vacuum condensates of neutrinos and quarks (denoted respectively with $w_\Lambda^{\nu-mix}$ and w_Λ^{q-mix}) imply contributions to the vacuum energy $\rho_\Lambda^{\nu-mix}$ and ρ_Λ^{q-mix} that are compatible with the estimated upper bound on the dark energy.

The constraint on the cut-off is imposed by the very small breaking of the Lorentz invariance of the flavor vacuum at the present epoch. Indeed, by solving numerically the equations for T_Λ^{mix} and V_Λ^{mix} , given respectively by Eqs.(8) and (9), we find that, in order to satisfy the condition $T_\Lambda^{mix} \ll V_\Lambda^{mix}$, due to the very small breaking of the Lorentz invariance, the cut-off on the momenta at the present epoch must be chosen such that

$$K_\Lambda \ll \sqrt[3]{m_1 m_2 m_3}. \quad (16)$$

In particular, the exact value of the adiabatic index of the vacuum mixing condensates (of neutrinos and quarks) at the present epoch, tells us how much K_Λ must be smaller than $\sqrt[3]{m_1 m_2 m_3}$.

In the order to derive an expression of the state equation of the vacuum mixing condensates as function of K_Λ , let us consider the adiabatic expansion of a sphere of volume V . Let p denote the pressure at which the sphere expands. The total energy, $E = \rho V$, is not conserved since the pressure does work. Assuming that temperature and number of particles are constant, according to the first law of thermodynamics, the work done by p must be equal to the change in the total energy: $dE = -p dV$. That is $\rho dV + V dp = -p dV$, that can be written as

$$d[(\rho + p)V] = 0, \quad (17)$$

from which

$$\rho + p = \frac{const}{V}. \quad (18)$$

Then the equation of state into the sphere can be written as

$$w = \frac{p}{\rho} = \frac{p}{\frac{const}{V} - p} = \frac{1}{\frac{C}{V} - 1}, \quad (19)$$

where C is a new constant. Note that $w \rightarrow -1$ if the volume is very large ($V \rightarrow \infty$), that is, in the bulk of the Universe, i.e. far from the Universe “boundaries”. In collapsed regions ($V \rightarrow 0$) we have $w \rightarrow 0$.

Eqs.(17)-(19) hold for any fluid contained in an expanding volume V , when entropy, temperature, number of particles and electrochemical potential are assumed constant and $p \approx const$. Considering then the flavor vacuum condensate at the present epoch, taking into account the conditions: $T_\Lambda^{mix} \ll V_\Lambda^{mix}$, and $\rho_\Lambda^{mix} \simeq -p_\Lambda^{mix} \simeq V_\Lambda^{mix}$, from Eqs.(7) and (18) we have respectively $\rho_\Lambda^{mix} = T_\Lambda^{mix} + V_\Lambda^{mix} \simeq V_\Lambda^{mix}$ and $\rho_\Lambda^{mix} = \frac{const}{V} - p_\Lambda^{mix}$. Thus the kinetic term is approximatively given by

$$T_\Lambda^{mix} \simeq \frac{const}{V}, \quad (20)$$

which means that, at the present epoch, the expansion of the universe leads to a smaller and smaller flavor vacuum condensate kinetic term. By using Eq.(20), the state equation for the flavor vacuum mixing condensate can be written as

$$w_\Lambda^{mix} = \frac{p_\Lambda^{mix}}{T_\Lambda^{mix} - p_\Lambda^{mix}}. \quad (21)$$

Eq.(21) shows that, since at the present epoch $T_\Lambda^{mix} \rightarrow 0$, then $w_\Lambda^{mix} \rightarrow -1$. Moreover, since T_Λ^{mix} and p_Λ^{mix} are function of the cut-off on the momenta K_Λ , then Eq.(21) gives an expression of w_Λ^{mix} as function of K_Λ : $w_\Lambda^{mix} = w_\Lambda^{mix}(K_\Lambda)$. We now estimate the contributions given to the dark energy by the particle mixing condensates for different values of w_Λ^{mix} close to -1 , both for neutrino and for quark mixing condensates.

A. Neutrino mixing condensate contribution

Let Ψ_f in Eq.(A2) represents the flavor neutrino fields: $\Psi_f^T = (\nu_e, \nu_\mu, \nu_\tau)$ and Ψ_m denotes the neutrino fields with definite masses, m_1, m_2, m_3 : $\Psi_m^T = (\nu_1, \nu_2, \nu_3)$. The experimental values of squared mass differences and mixing angles are respectively: $\Delta m_{12}^2 = 7.9 \times 10^{-5} eV^2$, $\Delta m_{23}^2 = 2.3 \times 10^{-3} eV^2$, $s_{12}^2 = 0.31$, $s_{23}^2 = 0.44$, $s_{13}^2 = 0.009$ [27]. In the normal hierarchy case: $|m_3| \gg |m_{1,2}|$, we consider values of the neutrino masses such that the experimental values of squared mass difference are satisfied, as for example: $m_1 = 4.6 \times 10^{-3} eV$, $m_2 = 1 \times 10^{-2} eV$, $m_3 = 5 \times 10^{-2} eV$. Then the condition Eq.(16) for neutrinos reads

$$K_\Lambda \ll 1.2 \times 10^{-2} eV. \quad (22)$$

In Table 1, we report the contribution of the neutrino mixing to the dark energy $\rho_\Lambda^{\nu-mix}$ and the corresponding state equation for different cut-offs satisfying the condition (22).

K_Λ	$\rho_\Lambda^{\nu-mix} (GeV^4)$	$T_\Lambda^{\nu-mix} (GeV^4)$	$w_\Lambda^{\nu-mix}$
$1.2 \times 10^{-2} eV$	1.1×10^{-45}	1.6×10^{-46}	-0.85
$4 \times 10^{-3} eV$	1.2×10^{-47}	3.5×10^{-49}	-0.97
$3 \times 10^{-3} eV$	0.3×10^{-47}	5.8×10^{-50}	-0.98
$4 \times 10^{-4} eV$	1.6×10^{-52}	6.1×10^{-56}	-0.99
$4 \times 10^{-5} eV$	1.6×10^{-57}	6.2×10^{-63}	-0.99

Table 1: Values of $\rho_\Lambda^{\nu-mix}$ and $w_\Lambda^{\nu-mix}$ for different cut-offs.

The result we find is that contributions to the dark energy compatible with its estimated upper bound: $\rho_\Lambda^{\nu-mix} \sim 10^{-47} GeV^4$ are obtained for values of the adiabatic index $w_\Lambda^{\nu-mix}$ of the neutrino mixing dark energy component:

$$-0.98 \leq w_\Lambda^{\nu-mix} \leq -0.97. \quad (23)$$

Eq.(23) is in agreement with the constraint on the equation of state of the dark energy given by the combination of WMAP and Supernova Legacy Survey (SNLS) data: $w = -0.967_{-0.072}^{+0.073}$ and with the constraint given by combining WMAP, large-scale structure and supernova data: $w = -1.08 \pm 0.12$ [28].

A value of $w_\Lambda^{\nu-mix} < -0.98$ leads to negligible contributions of $\rho_\Lambda^{\nu-mix}$. The results we found are dependent on the neutrino mass values one uses.

B. Quark mixing condensate contribution

The quark mixing is expressed as:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (24)$$

where $V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$ is the CKM matrix [29]. In such a case, in Eq.(A2), $\Psi_m^T = (d, s, b)$ and $\Psi_f^T = (d', s', b')$.

For the values of the quark masses given in Ref.[29], the condition Eq.(16) for quarks reads

$$K_\Lambda \ll 120 MeV. \quad (25)$$

In Table 2, we report the contribution of the quark mixing to the dark energy ρ_Λ^{q-mix} and the corresponding state equation for different cut-offs satisfying the condition (25).

K_Λ	$\rho_\Lambda^{q-mix} (GeV^4)$	$T_\Lambda^{q-mix} (GeV^4)$	w_Λ^{q-mix}
$120 MeV$	5.1×10^{-7}	3.5×10^{-7}	-0.3
$10 MeV$	2×10^{-10}	1.4×10^{-11}	-0.93
$300 KeV$	1.5×10^{-17}	1.8×10^{-21}	-0.99
$30 KeV$	1.5×10^{-22}	1.8×10^{-28}	-0.99
$0.3 eV$	1.5×10^{-47}	1.8×10^{-63}	-1

Table 2: Values of ρ_Λ^{q-mix} and w_Λ^{q-mix} for different cut-offs.

From Table 2, we find that the exact Lorentz invariance of the quark mixing condensate $w_\Lambda^{q-mix} = -1$ (T_Λ^{q-mix} is 16 orders less than V_Λ^{q-mix}), at the present epoch, leads to a contribution to the dark energy that is compatible with its estimated upper bound: $\rho_\Lambda^{q-mix} = 1.5 \times 10^{-47} GeV^4$. We remark that very small deviations from the value $w_\Lambda^{q-mix} = -1$ give rise to contributions of ρ_Λ^{q-mix} that are beyond the accepted upper bound of the dark energy.

The computation of ρ_Λ^{mix} turns out to be sensible to small variations in the values of the particle masses and of Δm^2 . Our results are therefore dependent on the mass values one uses.

It is our future plan to compare the present approach with the one of Ref.[30] based on string models of D-particle foam.

In conclusion, we have shown that under reasonable boundary conditions the vacuum condensate from particle mixing can provide contributions to the dark energy compatible with the observed value of the cosmological constant. At the present stage, the novelty of the mechanism here proposed in the study of dark energy and the remarkable improvement in the computed order of magnitude without needs of postulating (till now unobserved) exotic fields, reveals that the QFT particle mixing scenario provides an interesting approach to the dark energy problem.

IV. CONCLUSIONS AND DISCUSSION

We have shown that the energy density due to the vacuum condensate arising from the particle mixing can be interpreted as an evolving dark energy that at present epoch has a behavior and a value compatible with the observed cosmological constant. This value is obtained by imposing values of the adiabatic index close to -1 , both for vacuum condensates of neutrinos and quarks. Our discussion has been limited to the case of Minkowski space-time. In a forthcoming paper we will present the explicit computation in curved space-time. There we will show that the mixing treatment here presented in the flat space-time is a good approximation in the present epoch of that in FRW space-time.

A very short summary of the observational status of art can aid to clarify the frame for our considerations and results. As mentioned in the Introduction, the data accumulated in recent years indicate that the universe is dominated by a non-clustered fluid with negative pressure (the *dark energy*) able to drive the accelerated expansion. As a tentative solution to the inadequacy of the mentioned Λ CDM model, many authors have replaced the cosmological constant with a scalar field rolling down its potential and giving rise to models referred to as *quintessence* [31]. Even if successful in fitting the data, the quintessence approach to dark energy is still plagued by the coincidence problem since the dark energy and matter densities evolve differently and reach comparable values for a very limited portion of the universe evolution coinciding at present era. In this case, the coincidence problem is replaced with a fine-tuning problem. Moreover, it is not clear where this scalar field originates from, thus leaving a great uncertainty on the choice of the scalar field potential. The subtle and elusive nature of dark energy has led to look for completely different scenarios able to give a quintessential behavior without the need of exotic components. In this connection, it has been observed that the acceleration of the universe only claims for a negative pressure dominant component, but does not tell anything about the nature and the number of cosmic fluids filling the universe [32]. This consideration suggests that it could be possible to explain the accelerated expansion by introducing a single cosmic fluid with an equation of state causing it to act like dark matter at high densities (giving rise to clustered structures) and dark energy at low densities (then giving rise to accelerated behavior of cosmic fluid). An attractive feature of these models, usually referred to as *Unified Dark Energy* (UDE) or *Unified Dark Matter* (UDM) models, is that such an approach naturally solves, at least phenomenologically, the coincidence problem. Some interesting examples are the generalized Chaplygin gas [33], the tachyon field [34] and the condensate cosmology [35]. A different class of UDE models has been proposed [36] where a single fluid is considered whose energy density scales with the redshift in such a way that the radiation dominated era, the matter dominated era and the accelerating phase can be naturally achieved. Actually, there is still a different way to face the problem of cosmic acceleration. It is possible that the observed acceleration is not the manifestation of another ingredient in the cosmic pie, but rather the first signal of a breakdown of our understanding of the laws of gravitation [37, 38]. Examples of models comprising only the standard matter are provided by the Cardassian expansion [39], the DGP gravity [40], higher order gravity actions [41], non-vanishing torsion field [42], higher-order curvature invariants included in the gravity Lagrangian [43], etc..

This abundance of models is from one hand the signal of the fact that we have a limited number of cosmological tests to discriminate among rival theories, and from the other hand, that a urgent degeneracy problem has to be faced. The fact that the vacuum condensate originated by particle mixing provides contributions to the dark energy compatible with today expected value could contribute towards a solution of such a problem from both experimental and theoretical viewpoints.

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Appendix A: PARTICLE MIXING IN QUANTUM FIELD THEORY

The main features of the QFT formalism for the fermion mixing are here summarized (see [17] for a detailed review). The Lagrangian density describing three Dirac fields with a mixed mass term is:

$$\mathcal{L}(x) = \bar{\Psi}_f(x) (i \not{\partial} - \mathbf{M}) \Psi_f(x), \quad (\text{A1})$$

where $\Psi_f^T = (\psi_A, \psi_B, \psi_C)$ are the fields with definite flavors, and $\mathbf{M} = \mathbf{M}^\dagger$ is the mixed mass matrix. Among the various possible parameterizations of the mixing matrix for three fields, we work with CKM matrix of the form:

$$\Psi_f(x) = \mathcal{U} \Psi_m(x) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \Psi_m(x), \quad (\text{A2})$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, being θ_{ij} the mixing angle between ψ_i, ψ_j , δ is the CP violating phase and $\Psi_m^T = (\psi_1, \psi_2, \psi_3)$ are the fields with definite masses $m_1 \neq m_2 \neq m_3$:

$$\psi_i(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}, r} \left[u_{\mathbf{k}, i}^r \alpha_{\mathbf{k}, i}^r(t) + v_{-\mathbf{k}, i}^r \beta_{-\mathbf{k}, i}^{r\dagger}(t) \right] e^{i\mathbf{k} \cdot \mathbf{x}}, \quad i = 1, 2, 3, \quad (\text{A3})$$

with $\alpha_{\mathbf{k}, i}^r(t) = \alpha_{\mathbf{k}, i}^r e^{-i\omega_{k,i}t}$, $\beta_{\mathbf{k}, i}^{r\dagger}(t) = \beta_{\mathbf{k}, i}^{r\dagger} e^{i\omega_{k,i}t}$, and $\omega_{k,i} = \sqrt{\mathbf{k}^2 + m_i^2}$. The operators $\alpha_{\mathbf{k}, i}^r$ and $\beta_{\mathbf{k}, i}^r$, $i = 1, 2, 3$, $r = 1, 2$, annihilate the vacuum state $|0\rangle_m \equiv |0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3$: $\alpha_{\mathbf{k}, i}^r |0\rangle_m = \beta_{\mathbf{k}, i}^r |0\rangle_m = 0$. The anticommutation relations are: $\left\{ \nu_i^\alpha(x), \nu_j^{\beta\dagger}(y) \right\}_{t=t'} = \delta^3(\mathbf{x} - \mathbf{y}) \delta_{\alpha\beta} \delta_{ij}$, with $\alpha, \beta = 1, \dots, 4$, and $\left\{ \alpha_{\mathbf{k}, i}^r, \alpha_{\mathbf{q}, j}^{s\dagger} \right\} = \delta_{\mathbf{k}\mathbf{q}} \delta_{rs} \delta_{ij}$; $\left\{ \beta_{\mathbf{k}, i}^r, \beta_{\mathbf{q}, j}^{s\dagger} \right\} = \delta_{\mathbf{k}\mathbf{q}} \delta_{rs} \delta_{ij}$, with $i, j = 1, 2, 3$. All other anticommutators are zero. The orthonormality and completeness relations are: $u_{\mathbf{k}, i}^{r\dagger} u_{\mathbf{k}, i}^s = v_{\mathbf{k}, i}^{r\dagger} v_{\mathbf{k}, i}^s = \delta_{rs}$, $u_{\mathbf{k}, i}^{r\dagger} v_{-\mathbf{k}, i}^s = v_{-\mathbf{k}, i}^{r\dagger} u_{\mathbf{k}, i}^s = 0$, and $\sum_r (u_{\mathbf{k}, i}^r u_{\mathbf{k}, i}^{r\dagger} + v_{-\mathbf{k}, i}^r v_{-\mathbf{k}, i}^{r\dagger}) = 1$. Using Eq.(A2), we diagonalize the quadratic form of Eq.(A1), which then reduces to the Lagrangian for three Dirac fields, with masses m_1, m_2 and m_3 :

$$\mathcal{L}(x) = \bar{\Psi}_m(x) (i \not{\partial} - \mathbf{M}_d) \Psi_m(x), \quad (\text{A4})$$

where $\mathbf{M}_d = \text{diag}(m_1, m_2, m_3)$.

The mixing transformation can be written as $\psi_\sigma^\alpha(x) \equiv G_\theta^{-1}(t) \psi_i^\alpha(x) G_\theta(t)$, where $(\sigma, i) = (A, 1), (B, 2), (C, 3)$, and the generator is now

$$G_\theta(t) = G_{23}(t) G_{13}(t) G_{12}(t), \quad (\text{A5})$$

where

$$G_{12}(t) \equiv \exp \left[\theta_{12} \int d^3x \left(\psi_1^\dagger(x) \psi_2(x) - \psi_2^\dagger(x) \psi_1(x) \right) \right], \quad (\text{A6})$$

$$G_{23}(t) \equiv \exp \left[\theta_{23} \int d^3x \left(\psi_2^\dagger(x) \psi_3(x) - \psi_3^\dagger(x) \psi_2(x) \right) \right], \quad (\text{A7})$$

$$G_{13}(t) \equiv \exp \left[\theta_{13} \int d^3x \left(\psi_1^\dagger(x) \psi_3(x) e^{-i\delta} - \psi_3^\dagger(x) \psi_1(x) e^{i\delta} \right) \right]. \quad (\text{A8})$$

At finite volume, $G_\theta(t)$ is an unitary operator, $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^\dagger(t)$, preserving the canonical anticommutation relations; $G_\theta^{-1}(t)$ maps the Hilbert spaces for ψ_1, ψ_2 and ψ_3 fields \mathcal{H}_m to the Hilbert spaces for flavored fields \mathcal{H}_f : $G_\theta^{-1}(t) : \mathcal{H}_m \mapsto \mathcal{H}_f$. In particular, for the vacuum $|0\rangle_m$ we have, at finite volume V :

$$|0(t)\rangle_f = G_\theta^{-1}(t) |0\rangle_m. \quad (\text{A9})$$

$|0\rangle_f$ is the vacuum for \mathcal{H}_f , which we will refer to as the flavor vacuum. In the infinite volume limit the flavor vacuum $|0(t)\rangle_f$ turns out to be unitarily inequivalent to the vacuum for the massive neutrinos $|0\rangle_m$ [12]. This can be proved

for any number of generations by using rigorous mathematical methods [15]. The non-perturbative nature of the flavored vacuum for the mixed neutrinos is thus revealed.

Due to the linearity of $G_\theta(t)$, we can express the flavor annihilators, relative to the fields $\psi_\sigma(x)$ at each time, as (we use $(\sigma, i) = (A, 1), (B, 2), (C, 3)$):

$$\begin{aligned}\alpha_{\mathbf{k},\sigma}^r(t) &\equiv G_\theta^{-1}(t) \alpha_{\mathbf{k},i}^r(t) G_\theta(t), \\ \beta_{\mathbf{k},\sigma}^r(t) &\equiv G_\theta^{-1}(t) \beta_{\mathbf{k},i}^r(t) G_\theta(t).\end{aligned}\tag{A10}$$

The flavor fields can be expanded in the same bases as ν_i :

$$\psi_\sigma(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}, r} e^{i\mathbf{k} \cdot \mathbf{x}} \left[u_{\mathbf{k},i}^r \alpha_{\mathbf{k},\sigma}^r(t) + v_{-\mathbf{k},i}^r \beta_{-\mathbf{k},\sigma}^{r\dagger}(t) \right]. \tag{A11}$$

The flavor annihilation operators in the reference frame such that $\mathbf{k} = (0, 0, |\mathbf{k}|)$ are:

$$\alpha_{\mathbf{k},A}^r(t) = c_{12}c_{13} \alpha_{\mathbf{k},1}^r(t) + s_{12}c_{13} \left(|U_{12}^{\mathbf{k}}| \alpha_{\mathbf{k},2}^r(t) + \epsilon^r |V_{12}^{\mathbf{k}}| \beta_{-\mathbf{k},2}^{r\dagger}(t) \right) + e^{-i\delta} s_{13} \left(|U_{13}^{\mathbf{k}}| \alpha_{\mathbf{k},3}^r(t) + \epsilon^r |V_{13}^{\mathbf{k}}| \beta_{-\mathbf{k},3}^{r\dagger}(t) \right), \tag{A12}$$

$$\begin{aligned}\alpha_{\mathbf{k},B}^r(t) &= (c_{12}c_{23} - e^{i\delta} s_{12}s_{23}s_{13}) \alpha_{\mathbf{k},2}^r(t) - (s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}) \left(|U_{12}^{\mathbf{k}}| \alpha_{\mathbf{k},1}^r(t) - \epsilon^r |V_{12}^{\mathbf{k}}| \beta_{-\mathbf{k},1}^{r\dagger}(t) \right) \\ &\quad + s_{23}c_{13} \left(|U_{23}^{\mathbf{k}}| \alpha_{\mathbf{k},3}^r(t) + \epsilon^r |V_{23}^{\mathbf{k}}| \beta_{-\mathbf{k},3}^{r\dagger}(t) \right),\end{aligned}\tag{A13}$$

$$\begin{aligned}\alpha_{\mathbf{k},C}^r(t) &= c_{23}c_{13} \alpha_{\mathbf{k},3}^r(t) - (c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}) \left(|U_{23}^{\mathbf{k}}| \alpha_{\mathbf{k},2}^r(t) - \epsilon^r |V_{23}^{\mathbf{k}}| \beta_{-\mathbf{k},2}^{r\dagger}(t) \right) \\ &\quad + (s_{12}s_{23} - e^{i\delta} c_{12}c_{23}s_{13}) \left(|U_{13}^{\mathbf{k}}| \alpha_{\mathbf{k},1}^r(t) - \epsilon^r |V_{13}^{\mathbf{k}}| \beta_{-\mathbf{k},1}^{r\dagger}(t) \right),\end{aligned}\tag{A14}$$

$$\beta_{-\mathbf{k},A}^r(t) = c_{12}c_{13} \beta_{-\mathbf{k},1}^r(t) + s_{12}c_{13} \left(|U_{12}^{\mathbf{k}}| \beta_{-\mathbf{k},2}^r(t) - \epsilon^r |V_{12}^{\mathbf{k}}| \alpha_{\mathbf{k},2}^{r\dagger}(t) \right) + e^{i\delta} s_{13} \left(|U_{13}^{\mathbf{k}}| \beta_{-\mathbf{k},3}^r(t) - \epsilon^r |V_{13}^{\mathbf{k}}| \alpha_{\mathbf{k},3}^{r\dagger}(t) \right), \tag{A15}$$

$$\begin{aligned}\beta_{-\mathbf{k},B}^r(t) &= (c_{12}c_{23} - e^{-i\delta} s_{12}s_{23}s_{13}) \beta_{-\mathbf{k},2}^r(t) - (s_{12}c_{23} + e^{-i\delta} c_{12}s_{23}s_{13}) \left(|U_{12}^{\mathbf{k}}| \beta_{-\mathbf{k},1}^r(t) + \epsilon^r |V_{12}^{\mathbf{k}}| \alpha_{\mathbf{k},1}^{r\dagger}(t) \right) \\ &\quad + s_{23}c_{13} \left(|U_{23}^{\mathbf{k}}| \beta_{-\mathbf{k},3}^r(t) - \epsilon^r |V_{23}^{\mathbf{k}}| \alpha_{\mathbf{k},3}^{r\dagger}(t) \right),\end{aligned}\tag{A16}$$

$$\begin{aligned}\beta_{-\mathbf{k},C}^r(t) &= c_{23}c_{13} \beta_{-\mathbf{k},3}^r(t) - (c_{12}s_{23} + e^{-i\delta} s_{12}c_{23}s_{13}) \left(|U_{23}^{\mathbf{k}}| \beta_{-\mathbf{k},2}^r(t) + \epsilon^r |V_{23}^{\mathbf{k}}| \alpha_{\mathbf{k},2}^{r\dagger}(t) \right) \\ &\quad + (s_{12}s_{23} - e^{-i\delta} c_{12}c_{23}s_{13}) \left(|U_{13}^{\mathbf{k}}| \beta_{-\mathbf{k},1}^r(t) + \epsilon^r |V_{13}^{\mathbf{k}}| \alpha_{\mathbf{k},1}^{r\dagger}(t) \right).\end{aligned}\tag{A17}$$

These operators satisfy canonical (anti)commutation relations at equal times. $U_{ij}^{\mathbf{k}}$ and $V_{ij}^{\mathbf{k}}$ are Bogoliubov coefficients defined as:

$$|U_{ij}^{\mathbf{k}}| = \left(\frac{\omega_{k,i} + m_i}{2\omega_{k,i}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,j} + m_j}{2\omega_{k,j}} \right)^{\frac{1}{2}} \left(1 + \frac{|\mathbf{k}|^2}{(\omega_{k,i} + m_i)(\omega_{k,j} + m_j)} \right), \tag{A18}$$

$$|V_{ij}^{\mathbf{k}}| = \left(\frac{\omega_{k,i} + m_i}{2\omega_{k,i}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,j} + m_j}{2\omega_{k,j}} \right)^{\frac{1}{2}} \left(\frac{|\mathbf{k}|}{(\omega_{k,j} + m_j)} - \frac{|\mathbf{k}|}{(\omega_{k,i} + m_i)} \right), \tag{A19}$$

$$|U_{ij}^{\mathbf{k}}|^2 + |V_{ij}^{\mathbf{k}}|^2 = 1, \tag{A20}$$

where $i, j = 1, 2, 3$ and $j > i$. The numbers of particles condensed in the vacuum are different for fermions of different masses:

$$\mathcal{N}_1^{\mathbf{k}} = {}_f\langle 0(t) | N_{\alpha_1}^{\mathbf{k},r} | 0(t) \rangle_f = {}_f\langle 0(t) | N_{\beta_1}^{\mathbf{k},r} | 0(t) \rangle_f = s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2, \tag{A21}$$

$$\mathcal{N}_2^{\mathbf{k}} = {}_f\langle 0(t) | N_{\alpha_2}^{\mathbf{k},r} | 0(t) \rangle_f = {}_f\langle 0(t) | N_{\beta_2}^{\mathbf{k},r} | 0(t) \rangle_f = |-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2, \tag{A22}$$

$$\mathcal{N}_3^{\mathbf{k}} = {}_f\langle 0(t) | N_{\alpha_3}^{\mathbf{k},r} | 0(t) \rangle_f = {}_f\langle 0(t) | N_{\beta_3}^{\mathbf{k},r} | 0(t) \rangle_f = |-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2. \tag{A23}$$

Since the vacuum $|0\rangle_m$ for the massive fields is unitarily inequivalent to the vacuum $|0(t)\rangle_f$ for the mixed (flavored) fields at time t , for any t , two different normal orderings must be defined, respectively with respect to $|0\rangle_m$, as usual denoted by $: \dots :$, and with respect to $|0(t)\rangle_f$, denoted by $:: \dots ::$. The Hamiltonian normal ordered with respect to the vacua $|0\rangle_m$ and $|0(t)\rangle_f$ are given respectively by

$$: H := H - {}_m\langle 0|H|0\rangle_m = H + 2 \int d^3\mathbf{k} (\omega_{k,1} + \omega_{k,2} + \omega_{k,3}) = \sum_i \sum_r \int d^3\mathbf{k} \omega_{k,i} [\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r], \quad (\text{A24})$$

$$\begin{aligned} :: H :: &\equiv H - {}_f\langle 0(t)|H|0(t)\rangle_f = H + 2 \int d^3\mathbf{k} (\omega_{k,1} + \omega_{k,2} + \omega_{k,3}) - 4 \int d^3\mathbf{k} \left[\omega_{k,1} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) \right. \\ &+ \omega_{k,2} \left(|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2 \right) \\ &\left. + \omega_{k,3} \left(|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2 \right) \right]. \end{aligned} \quad (\text{A25})$$

The state $|0(t)\rangle_f$ is a condensate of massive particle-antiparticle pairs. Note that the difference of energy between $|0(t)\rangle_f$ and $|0\rangle_m$ represents the energy of the condensed neutrinos given in Eqs.(A21)-(A23)

$$\begin{aligned} {}_f\langle 0(t)| : H : |0(t)\rangle_f &= {}_f\langle 0(t)|H|0(t)\rangle_f - {}_m\langle 0|H|0\rangle_m = 4 \int d^3\mathbf{k} \left[\omega_{k,1} (s_{12}^2 c_{13}^2 |V_{12}^{\mathbf{k}}|^2 + s_{13}^2 |V_{13}^{\mathbf{k}}|^2) \right. \\ &+ \omega_{k,2} \left(|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 |V_{12}^{\mathbf{k}}|^2 + s_{23}^2 c_{13}^2 |V_{23}^{\mathbf{k}}|^2 \right) \\ &\left. + \omega_{k,3} \left(|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 |V_{23}^{\mathbf{k}}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 |V_{13}^{\mathbf{k}}|^2 \right) \right]. \end{aligned} \quad (\text{A26})$$

Appendix B: BEHAVIOR OF ρ_Λ^{mix} FOR $K_\Lambda \gg m_1, m_2, m_3$

By solving Eq.(14), we obtain the explicit expression for ρ_Λ^{mix} , which for $K_\Lambda \gg m_1, m_2, m_3$ reduces to:

$$\begin{aligned} \rho_\Lambda^{mix} &\approx \frac{m_1^2}{2\pi} \left\{ \frac{2s_{13}^2 m_1^2 (m_3 - m_1)}{\sqrt{m_3^2 - m_1^2}} \arctan\left(\frac{\sqrt{m_3^2 - m_1^2}}{m_1}\right) + \frac{2s_{12}^2 c_{13}^2 m_1^2 (m_2 - m_1)}{\sqrt{m_2^2 - m_1^2}} \arctan\left(\frac{\sqrt{m_2^2 - m_1^2}}{m_1}\right) \right. \\ &+ s_{13}^2 (m_3^2 - 2m_3m_1 + 2m_1^2) \log\left(\frac{2K_\Lambda}{m_3}\right) - (s_{12}^2 c_{13}^2 + s_{13}^2) m_1^2 \log\left(\frac{2K_\Lambda}{m_1}\right) \\ &+ s_{12}^2 c_{13}^2 (m_2^2 - 2m_2m_1 + 2m_1^2) \log\left(\frac{2K_\Lambda}{m_2}\right) \left. \right\} + \frac{m_2^2}{2\pi} \left\{ \frac{2s_{23}^2 c_{13}^2 m_2^2 (m_3 - m_2)}{\sqrt{m_3^2 - m_2^2}} \arctan\left(\frac{\sqrt{m_3^2 - m_2^2}}{m_2}\right) \right. \\ &+ \frac{2 |-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 m_2^2 (m_2 - m_1)}{\sqrt{m_2^2 - m_1^2}} \tanh^{-1}\left(\frac{\sqrt{m_2^2 - m_1^2}}{m_2}\right) \\ &+ s_{23}^2 c_{13}^2 (m_3^2 - 2m_3m_2 + 2m_2^2) \log\left(\frac{2K_\Lambda}{m_3}\right) + (|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 + s_{23}^2 c_{13}^2) m_2^2 \log\left(\frac{2K_\Lambda}{m_2}\right) \\ &+ |-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}|^2 (m_1^2 - 2m_2m_1 + 2m_2^2) \log\left(\frac{2K_\Lambda}{m_2}\right) \left. \right\} \\ &+ \frac{m_3^2}{2\pi} \left\{ \frac{2 |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 m_3^2 (m_3 - m_1)}{\sqrt{m_3^2 - m_1^2}} \tanh^{-1}\left(\frac{\sqrt{m_3^2 - m_1^2}}{m_3}\right) \right. \\ &+ \frac{2 |-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 m_3^2 (m_3 - m_2)}{\sqrt{m_3^2 - m_2^2}} \tanh^{-1}\left(\frac{\sqrt{m_3^2 - m_2^2}}{m_3}\right) \\ &+ |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2 (2m_3^2 - 2m_3m_1 + m_1^2) \log\left(\frac{2K_\Lambda}{m_1}\right) \\ &+ (|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 + |s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}|^2) m_3^2 \log\left(\frac{2K_\Lambda}{m_3}\right) \\ &\left. + |-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}|^2 (m_2^2 - 2m_3m_2 + 2m_3^2) \log\left(\frac{2K_\Lambda}{m_2}\right) \right\}. \end{aligned} \quad (\text{B1})$$

This quantity diverges in K_Λ as $m_i^4 \log(2K_\Lambda/m_j)$, with $i, j = 1, 2, 3$.

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